Theoretical paper

Notebook: myao2018

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Location: 5°21'30 N 100°18'18 E

Theoretical

(1) We use Stefan - Boltzmann law

$$E = \sigma T^4 \ \text{j} - \text{m}^{-2} - \text{s}^{-1} \qquad (40)$$

where E: the total energy radiated per unit ~~area~~ surface area of a black body across all wavelengths per unit time

σ: Stefan - Boltzmann constant

$= 5.670373 \times 10^{-8}\, W - m^{-2} - K^{-4}$

For the photosphere, we have

$$E_1 = \sigma T_1^4 = \sigma (5,800\,K)^4 \qquad (20)$$

For the sunspot, we have

$$E_2 = \sigma T_2^4 = \sigma (4,200\,K)^4 \qquad (20)$$

$\therefore$ each square meter of the photosphere is brighter than each square meter of the sunspot by

$$\frac{E_1}{E_2} = \frac{\sigma T_1^4}{\sigma T_2^4} = \left(\frac{5,800\,K}{4,200\,K}\right)^4 = 3.636 \text{ times}$$

(20)

(2) We use Pogson's Equation

$$M = m - 5\log_{10} d + 5 \qquad (40)$$

where M: absolute magnitude of star
m: apparent magnitude of star
d: distance of star from Sun in parsecs

(a) We have, for the first star,

$$M = m_1 - 5\log_{10} d_1 + 5 \quad \text{— ①}$$

and for the second star,

$$M = m_2 - 5\log_{10} d_2 + 5 \quad \text{— ②} \quad (20)$$

and $\frac{d_1}{d_2} = 1{,}500$

$$\therefore \quad m_1 - 5\log_{10} d_1 + 5 = m_2 - 5\log_{10} d_2 + 5$$

$$m_1 - m_2 = \log_{10}\left(\frac{d_1}{d_2}\right)^5 = \log_{10}(1{,}500)^5$$

$$\doteq 15.88 \quad (20)$$

(b) The larger apparent magnitude is $m_1 = 15.88 + m_2$ for the star that is further away

(20)

(3)

(a) From Kepler's Third Law

$$T^2 \propto r^3$$
$$= \frac{4\pi^2 r^3}{GM} \qquad (40)$$

where T: period around the Sun
r: semi-major axis of space probe in its orbit around the Sun
G: universal gravitational constant
M: mass of Sun

We also have, $r = \frac{r_p + r_a}{2}$ (20)

where r_p: perihelion distance
r_a: aphelion distance

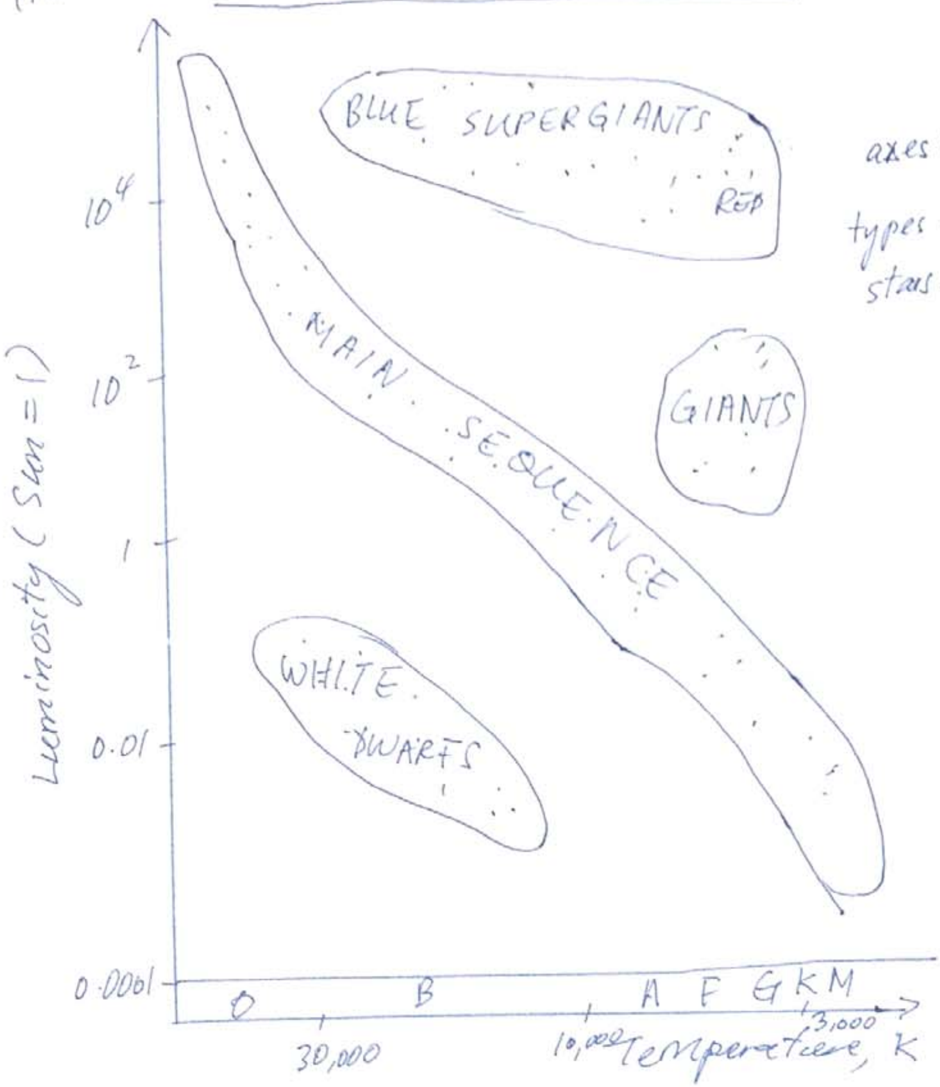
$$r = \frac{0.4AU + 5.4AU}{2} = 2.9\ AU$$
$$= 2.9 \times 1.496 \times 10^8\ km$$
$$= 4.338 \times 10^{11}\ m$$

$$\therefore \quad T^2 = \frac{4\pi^2 (4.338 \times 10^{11} m)^3}{(6.674 \times 10^{-11} m^3 kg^{-1} s^{-2})(1.989 \times 10^{30} kg)}$$
$$= 2.428 \times 10^{16}\ s^2$$

and $T = \sqrt{2.428 \times 10^{16} s^2} = 155,835,718\ s$
$$= \frac{155,835,718}{3600 \times 24 \times 365.25}$$
$$= 4.938\ years \qquad (20)$$

(b) Since the space probe is in orbit around the Sun, its orbit cannot be a parabola or hyperbola
Since its orbit has a perihelion and a aphelion, its orbit cannot be a circle and is therefore an ellipse.
(20)

(4) Hertzsprung – Russell Diagram



axes: (30)

types of stars: (70)

5) The planet Mars was at opposition with Earth at UTC 13 hours 13 minutes on 27th July 2018. Each Mars opposition is separated from the next opposition by ~~778~~ days, or 2 years, 1 month and 18 days. Therefore the next Mars opposition will occur at (50)

~~13 October 2020~~

2320 UTC on 13 October 2020 (30)

~~100~~

(50)

$$\frac{1}{P_{syn}} = \frac{1}{P_e} - \frac{1}{P_m} \qquad (40)$$

$$= \frac{1}{365.26} - \frac{1}{686.97}$$

$$= 779.965 \text{ Earth days} \qquad (30)$$

The Schwarzschild Radius, r_s, is given by

$$r_s = \frac{2GM}{c^2} \qquad (40)$$

where G: universal gravitational constant

M: mass of supermassive black hole

$= 4 \times 10^6 M_\odot$

where $M_\odot$ is the mass of the Sun

c: speed of light

$\therefore$ ~~$r_s = 2(6.674$~~

$$\therefore r_s = \frac{2(6.674 \times 10^{-11} m^3 kg^{-1} s^{-2})(4 \times 10^6 \times 1.989 \times 10^{30} kg)}{(2.9979 \times 10^8 m\text{-}s^{-1})^2}$$

$$= 1.182 \times 10^{10} m \qquad (60)$$

$$= 11.82 \text{ billion km}$$

$\equiv$

7) (a) We use Pogson's Equation

$M = m - 5\log_{10} d + 5$ (20)

where M: absolute magnitude of star

m: apparent magnitude of star

d: distance of star from Sun in parsecs

Putting in the values, we have

$2.1 = 7.5 - 5\log_{10} d + 5$

Rearranging,

$d = 10^{2.08}$ parsecs

$= 120.226$ parsecs

$= 120.226 \times 3.26$ light-years

$= 391.938$ light years (30)

(b) The luminosity, L, of a star is the total amount of energy emitted per unit time by a star, measured in watts. (20)

$L = \sigma A T^4$

where σ: Stefan-Boltzmann constant

A: total surface area of star

T: absolute temperature of surface of star

$= (A)(\sigma T^4) = (A)(5.670 \times 10^{-8})(9{,}500K)^4$

$= (A\ m^2)(4.618 \times 10^8$ watts$)$

$= (A)(4.618 \times 10^8)$ joules (30)

(8) We use the Rayleigh Criterion for the angular resolution, $\Delta\theta$,

$$\Delta\theta \text{ (radians)} = \frac{1.22\lambda}{D} \quad \text{— ①} \quad (20)$$

where λ: wavelength

D: diameter of optical telescope

Also, we have

$$s = l(\Delta\theta) \quad \text{— ②} \quad (20)$$

where s: diameter of moon crater

l: distance from Moon to Earth

$\Delta\theta$: angular resolution of telescope

From Equation ②, we have

$$\Delta\theta = \frac{2 \times 10^{3} \text{ m}}{384,400 \times 10^{3} \text{ m}} = 5.203 \times 10^{-6} \text{ radian} \quad 20$$

Putting this answer into Equation ①,

$$5.203 \times 10^{-6} \text{ radian} = \frac{1.22 \times 555 \times 10^{-9} \text{ m}}{D} \quad 20$$

(555 nm is the peak of eye sensitivity in the visible light region)

$$\therefore \quad D = \frac{1.22 \times 555 \times 10^{-9} \text{ m}}{5.203 \times 10^{-6} \text{ radian}}$$

$$= 0.1301 \text{ m}$$

$$\equiv 5.12 \text{ inches} \quad 20 \qquad (160)$$

To resolve the Moon crater, we need a telescope of at least 0.1301 m diameter

b) We use the Hubble's Law

(40)

$$v = H_0 r$$

where v: recession velocity of galaxy as measured by redshift

H_0: Hubble constant

r: distance of galaxy to Sun.

Rearranging Hubble's Law,

$$\frac{1}{H_0} = \frac{r}{v} = \frac{\text{distance}}{\text{velocity}} = \text{time}$$

∴ by calculating the ~~rest~~ reciprocal of the Hubble constant we can estimate the Age of the Universe.

We have

$$\frac{1}{H_0} = \frac{1}{73.52 \text{ km-s}^{-1} (\text{Mpc})^{-1}}$$

$$= \frac{1}{\left(\frac{73.52 \times 10^3 \text{ m-s}^{-1}}{10^6 \times 3.26 \times 3 \times 10^8 \times 365.25 \times 24 \times 3600}\right)}$$

$$= \frac{1}{(2.3824 \times 10^{-18} \text{ s}^{-1})}$$ (60)

$$= 4.1974 \times 10^{17} \text{ s}$$

$$= \frac{4.1974 \times 10^{17} \text{ s}}{3600 \times 24 \times 365.25} = 1.330 \times 10^{10} \text{ years}$$

$$= 13.30 \text{ billion years}$$